ON THE DEFLECTION THEORY OF RIBBED TWO-HINGED ELASTIC ARCHES

BY

PARESH NATH CHATTERJEE

B.Sc., University of Calcutta, India, 1938
B.E., University of Calcutta, India, 1941
M.S., University of Illinois, 1947

R. UNIV. BIBLIOTHEEK LEIDEN

AN ABSTRACT OF A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING IN THE GRADUATE COLLEGE OF THE UNIVERSITY OF ILLINOIS, 1949

URBANA, ILLINOIS
1952

Acknowledgment

The writer is deeply indebted to Thomas C. Shedd, Professor of Structural Engineering at the University of Illinois, under whose direction this investigation was made and whose most valuable guidance inspired the writer all through the investigation. The writer also takes this opportunity to express his appreciation to others whose suggestions and criticisms helped in the development of this thesis.

ON THE DEFLECTION THEORY OF RIBBED TWO-HINGED ELASTIC ARCHES

1. General

In the analysis of two-hinged arches by the ordinary theory the effect of deflection on moment is neglected. In an elastic arch with a slender rib, however, this approximation may give rise to a considerable error (on the unsafe side), and a more refined procedure, "the deflection theory," is called for, in which the effect of deflection on moment is taken into account.

2. Existing Deflection Theories

Several well-known deflection theories proposed in the past by the various authors have been discussed in the thesis. The basic approach in these theories is fundamentally the same and they are based on that proposed by Melan in 1906. In all of them a differential equation for the vertical component of deflection is formed in terms of the bending moment and the horizontal thrust. The other equations necessary for a solution are then formed, first from considerations of statics, and second either from considerations of internal energy or from the fact that the total horizontal deflection over the entire span must be zero. The horizontal component of deflection is entirely neglected in all the theories.

The usefulness of the above theories in design is rather limited. Due to the algebraic nature of the procedures, a solution of the equations is only practicable for simplified conditions of the loading, shape of the arch axis, and variation of the moment of inertia. The most objectionable approximation, however, does not lie in the above simplifications but in the assumption that the horizontal component of deflection may be neglected. This approximation leads to computed moments less than actually exist, and in a two-hinged arch with high rise-span ratio the error may be considerable (for an arch with 1:2 rise-span ratio the error may be as high as 25%).

To correct for the objectionable neglect of the horizontal component of deflection a modification of the above theories has been proposed by the writer. The expression for the vertical component of deflection arrived at in existing theories, neglecting the horizontal component of deflection, may be taken to be correct, and may then be used to obtain an expression for the horizontal component of deflection. More accurate bending moments may then be calculated by taking into account the horizontal component of deflection obtained from the above procedure.

3. The Proposed Deflection Theory Based on a Numerical Procedure of Successive Approximation

To overcome the inherent difficulties encountered in the existing deflection theories, a numerical procedure based on successive approximations is suggested by the writer. The procedure is basically the same as the other deflection theories, except that the solution of the equations involved is sought by a process of successive approximations rather than directly as attempted in other theories. The vertical and the horizontal components of the deflection are expressed in the form of integral equations, instead of in the form of differential equations as has been done in existing theories. A set of deflections compatible with the boundary conditions are then assumed and the vertical and horizontal components of deflection are determined by numerical integration. The deflections obtained in this manner are a better approximation than those originally assumed. The process is repeated with the thus computed deflections assumed to satisfy boundary conditions, until a satisfactory agreement is reached between the assumed and the computed deflections. In the first approximation the computations are based on the undeflected axis and thus correspond to the ordinary theory of arch analysis. A procedure has also been suggested by which the true deflections and moments can be estimated successfully from the first approximation. A study by the writer showed that the deflections and moments estimated in this manner are sufficiently accurate for the purposes of design.

The main advantage of the proposed procedure lies in the versatility of its application. Because of the numerical nature of the solution, it is not necessary to make simplifying assumptions regarding load conditions, shape of the arch axis, or variation of the moment of inertia of the rib, which are so necessary in the older deflection theories. The horizontal component of deflection which had to be ignored in previous studies can be taken into account easily. Furthermore, computations may be so systematized that they are readily usable in a design office as a routine procedure.

4. Numerical Analysis of Cambered Two-Hinged Arches

In practice a two-hinged arch is often cambered for the dead load (and sometimes also for a portion of the live load) in such a manner that the arch acts as a three-hinged arch for the camber load, and as a two-hinged arch for any other loading. A numerical procedure has been devised by which the amount of camber necessary for this purpose can be determined by the deflection theory. A modification of the proposed

numerical procedure of arch analysis has also been suggested for analysing arches cambered in such a manner.

5. An Approximate Deflection Theory for Two-Hinged Arches

In the design of a two-hinged arch it is desirable to have an approximate procedure by which the preliminary design may be made without going through a complicated analysis. An approximate deflection theory has been developed by which the true deflections and moments may be estimated with sufficient accuracy from the deflections and moments obtained from the ordinary theory of arch analysis.

In the approximate deflection theory, the deflections and moments are expressed by the formulae:

$$\left.\begin{matrix}\dfrac{\eta_f}{\eta_o} \\ \dfrac{M_f}{M_o}\end{matrix}\right\} = \frac{1}{1 - H/H_{cr}},$$

in which

η_f is the deflection based on the approximate deflection theory,
η_o is the deflection obtained from the ordinary theory,
M_f is the bending moment based on the deflection theory,
M_o is the bending moment obtained from the ordinary theory,
H is the horizontal thrust and may be taken to be that determined from the ordinary theory of arch analysis,
H_{cr} is the critical horizontal thrust compatible with the deflection configuration of the arch axis.

For the usual loading condition for maximum moment at any point on the arch near one of the quarter points, the arch deflects in a two-looped configuration approximately antisymmetrical to the crown, and the lowest value of H_{cr} which corresponds to that deflection pattern should be used. For the loading condition that would produce maximum moment at or near to the crown, the arch deflects in a three-looped configuration and the second lowest value of H_{cr} which corresponds to that deflection configuration should be used.

A study of the lowest and the second lowest critical horizontal thrusts for two-hinged parabolic arches having constant moment of inertia and different rise-span ratio has been made in Appendix A of the thesis and the results are plotted in curves. To use the deflection or moment formula of the approximate deflection theory for parabolic or nearly parabolic arches with ribs having constant moment of inertia, the value of H_{cr} may be directly read from the curves. In an arch with

variable moment of inertia an equivalent moment of inertia assumed to be constant may be used by taking the average of the moment of inertia or better still by taking a weighted average of the moment of inertia. A simple calculation procedure to determine a suitably weighted average moment of inertia for a two-hinged arch rib has been suggested.

6. A Design Procedure by the Deflection Theory

Finally a design procedure using the deflection theory has been suggested. A simple procedure has been proposed by which the maximum quarter point moment, crown moment, and the corresponding horizontal thrusts and the maximum quarter point deflection may be determined approximately for the purpose of the preliminary proportioning of the arch rib. After the preliminary proportioning, the preliminary design may be made. The maximum bending moment and the corresponding horizontal thrust at any point may be determined by the ordinary theory of arch analysis from the usual influence line method, and may then be modified by the moment formula of the approximate deflection theory.

In the final design the first approximation of the proposed numerical procedure should be carried out for the loading condition that would produce maximum moment at any point on the arch axis. From the deflections obtained in this manner the true deflections, horizontal thrust and bending moment may be estimated from the procedure developed in the thesis with sufficient accuracy for design purposes. If greater accuracy is desired, successive approximations may be carried out starting from the estimated deflections.

7. Criterion for the Design by the Deflection Theory

A criterion has also been set up by which it may be decided whether, for a particular case, the more refined methods of the deflection theory are necessary. In an arch designed under such conditions that approximately half the total maximum boundary stress results from the axial thrust and the rest from the bending moment, a 10% increase in stress will result in a 5% increase in the maximum boundary stress. If 5% is accepted as the limit of tolerance in design, it may be stated that an arch should be designed by the more accurate deflection theory when the ratio H/H_{cr} is greater than 0.10, which would give approximately 10% increase in moments computed by the deflection theory as compared with those computed by the ordinary theory.

VITA

The writer is of Indian nationality and was born at Dacca, Bengal, India (now in Pakistan) on November 26, 1918. He matriculated from the Dinajpore Zilla High School, Bengal, India, in 1934, and entered the Presidency College of the University of Calcutta from which he obtained in 1938 the degree of Bachelor of Science, with Honours in Physics. He then entered the Bengal Engineering College of the University of Calcutta and obtained the degree of Bachelor of Engineering (Civil) in 1941.

He took practical training for one year (1942) as a student engineer at the Communication and Works Department of the Government of Bengal, which was a requirement for the degree in Engineering. In 1944 he joined the Britannia Building and Iron Company, a structural engineering firm at Calcutta as an Assistant Engineer, and was in charge of the extension of an aluminum rolling mill at Belur, Calcutta.

In 1945 he was awarded by the (Provincial) Government of Bengal, India, an overseas scholarship for higher education. He came to the United States in January 1946, and has been following his studies at the Graduate College of Civil Engineering of the University of Illinois since the second semester of 1945-46. He obtained the degree of Master of Science from the University of Illinois in 1947.

He is a member of the American Concrete Institute, and an associate member of Sigma Xi.